3 The Mathematics of Sharing

3.1 Fair-Division Games

3.2 Two Players: The Divider-Chooser Method

3.3 The Lone-Divider Method

3.4 The Lone-Chooser Method

3.5 The Last-Diminsher Method

3.6 The Method of Sealed Bids

3.7 The Method of Markers

Divider-Chooser Method

What? the best known of all continuous fair-division methods.

When? This method can be used anytime the fair-division game involves two players and a *continuous* set *S*

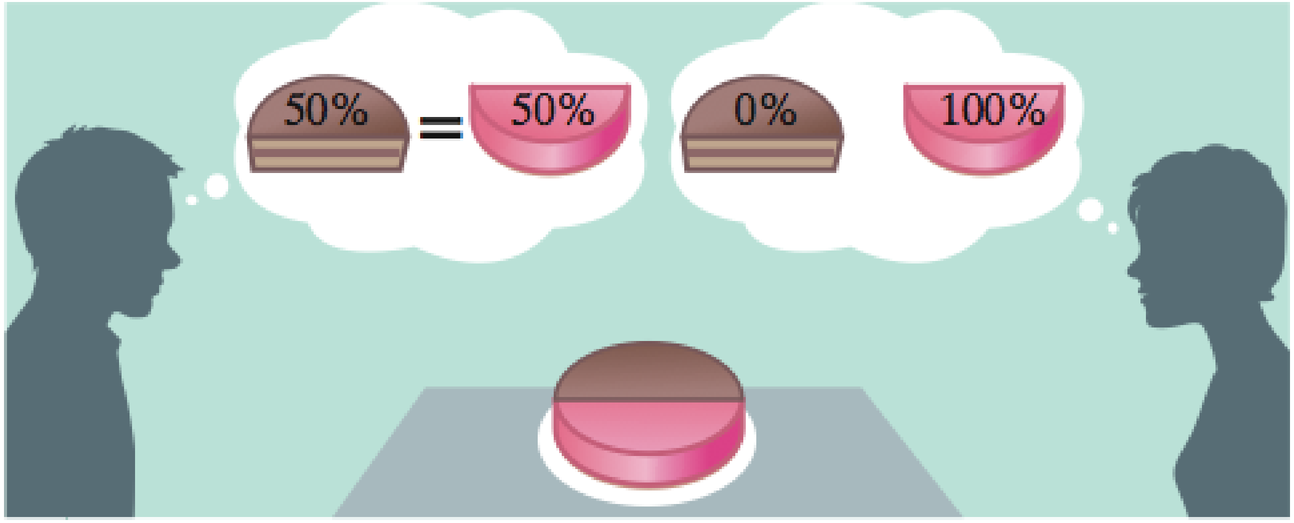
How? Informally it is best known as the *you cut–I choose method*. As this name suggests, one player, called the *divider*, divides *S* into two shares, and the second player, called the *chooser*, picks the share he or she wants, leaving the other share to the divider.

Divider-Chooser Method

Under the rationality and privacy assumptions we introduced in the previous section, this method guarantees that both divider and chooser will get a fair share

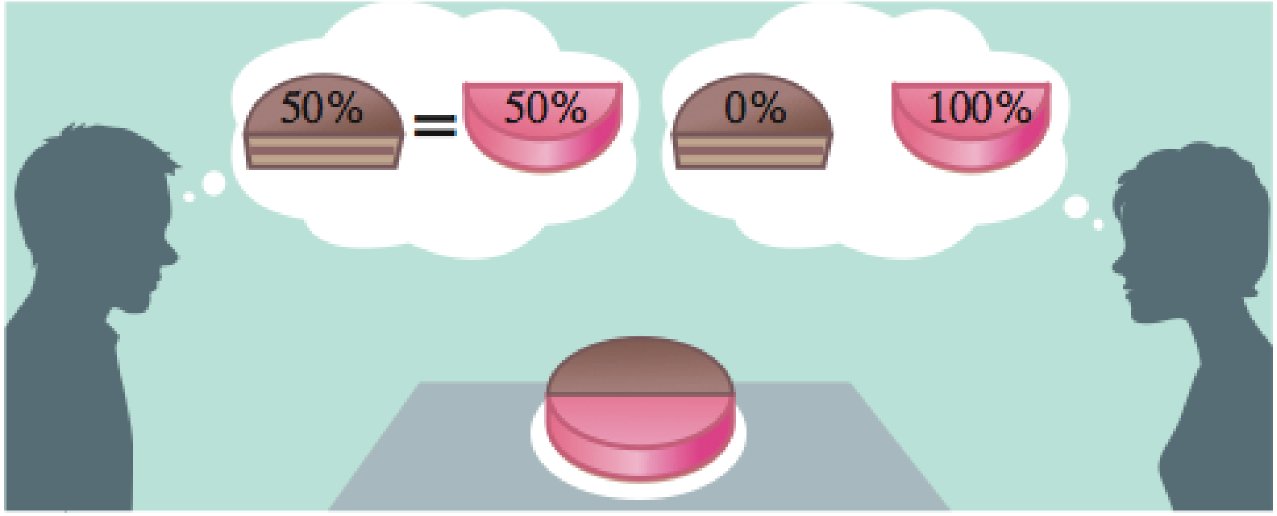
Example 3.1 Damian and Cleo Divide a Cheesecake

On their first date, Damian and Cleo go to the county fair. They buy jointly a raffle ticket, and as luck would have it, they win a half chocolate–half strawberry cheesecake. Damian likes chocolate and strawberry equally well, so in his eyes the chocolate and strawberry halves are equal in value.



Example 3.1 Damian and Cleo Divide a Cheesecake

On the other hand, Cleo hates chocolate–she is allergic to it and gets sick if she eats any–so in her eyes the value of the cake is 0% for the chocolate half, 100% for the strawberry part.



Once again, to ensure a fair division, we will assume that neither of them knows anything about the other's likes and dislikes.

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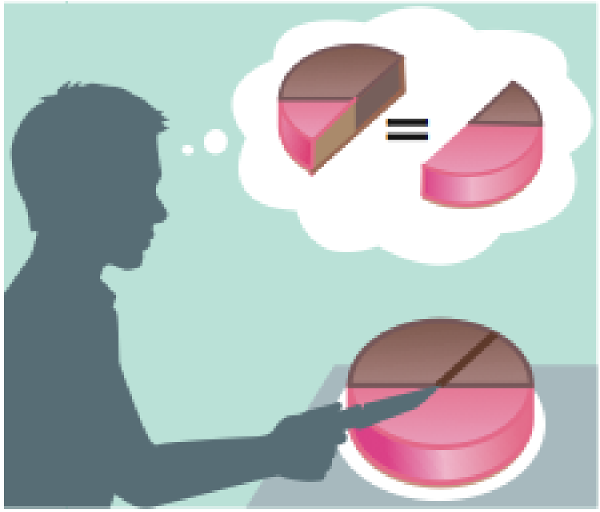
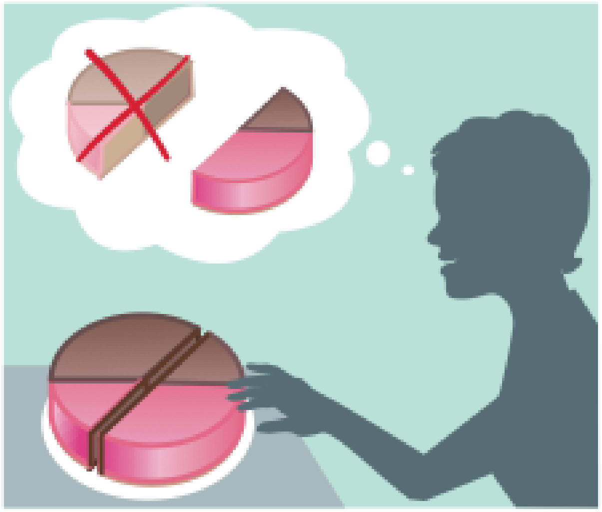
Damian volunteers to go first (the divider). He cuts the cake in a perfectly rational division of the cake based on his value system–each piece is half of the cake and to him worth one-half of the total value of the cake. It is now Cleo's turn to choose, and her choice is obvious she will pick the piece having the larger strawberry part.





Example 3.1 Damian and Cleo Divide a Cheesecake

The final outcome of this division is that Damian gets a piece that in his own eyes is worth exactly half of the cake, but Cleo ends up with a much sweeter deal–a piece that in her own eyes is worth about two-thirds of the cake. This is, nonetheless, a fair division of the cake–both players get pieces worth 50% or more.

Better to be the Chooser

Example 3.1 illustrates why, given a choice, it is always better to be the chooser than the divider–the divider is guaranteed a share worth exactly 50% of the total value of *S*, but with just a little luck the chooser can end up with a share worth more than 50%.

Since a fair-division method should treat all players equally, both players should have an equal chance of being the chooser. This is best done by means of a coin toss, with the winner of the coin toss getting the privilege of making the choice.