B Banzhaf Power

11. Consider the weighted voting system [10: 6, 5, 4, 2].

(a) What is the weight of the coalition formed by P_1 and P_3?

(b) Write down all winning coalitions.

(c) Which players are critical in the coalition $\{P_1, P_2, P_3\}$?

(d) Find the Banzhaf power distribution of this weighted voting system.

(a) $6+4 = \boxed{10}$

(b) $\{P_1, P_2\}$ $\{P_1, P_3\}$
$\{P_1, P_2, P_3\}$ $\{P_1, P_2, P_4\}$ $\{P_1, P_3, P_4\}$
$\{P_2, P_3, P_4\}$ $\{P_1, P_2, P_3, P_4\}$
↑ any pair or group that have 10 or more.

(c) P_1. If you take P_1 out, then the coalition cannot win.

Critical:
B_1: |||| B_2: ||| B_3: ||| B_4: |

(d) $\beta_1 = \frac{5}{12}$ $\beta_2 = \frac{3}{12}$ $\beta_3 = \frac{3}{12}$ $\beta_4 = \frac{1}{12}$

12. Consider the weighted voting system [5: 3, 2, 1, 1].

(a) What is the weight of the coalition formed by P_1 and P_3?

(b) Which players are critical in the coalition $\{P_1, P_2, P_3\}$?

(c) Which players are critical in the coalition $\{P_1, P_3, P_4\}$?

(d) Write down all winning coalitions.

(e) Find the Banzhaf power distribution of this weighted voting system.

(a) $\boxed{4}$

(b) P_1 and P_2

(c) P_1, P_3, and P_4

(d) $\{P_1, P_2\}$
$\{P_1, P_2, P_3\}$ $\{P_1, P_2, P_4\}$ $\{P_1, P_3, P_4\}$
$\{P_1, P_2, P_3, P_4\}$

Critical:
B_1: |||| B_2: ||| B_3: | B_4: |

(e) $\beta_1 = \frac{5}{10}$ $\beta_2 = \frac{3}{10}$ $\beta_3 = \frac{1}{10}$ $\beta_4 = \frac{1}{10}$

13. **(a)** Find the Banzhaf power distribution of the weighted voting system [6: 5, 2, 1].

(b) Find the Banzhaf power distribution of the weighted voting system [3: 2, 1, 1]. Compare your answers in (a) and (b).

(a) $\{P_1, P_2\}$ $\{P_1, P_3\}$ $\{P_1, P_2, P_3\}$

↳ B_1: 111 B_2: 1 B_3: 1 /5 T

$\beta_1 = 3/5$ $\beta_2 = 1/5$ $\beta_3 = 1/5$

(b) $\{P_1, P_2\}$ $\{P_1, P_3\}$ $\{P_1, P_2, P_3\}$

↳ B_1: 111 B_2: 1 B_3: 1

$\beta_1 = 3/5$ $\beta_2 = 1/5$ $\beta_3 = 1/5$ /5 T

...compare ... they have same distribution.

14. **(a)** Find the Banzhaf power distribution of the weighted voting system [7: 5, 2, 1].

(b) Find the Banzhaf power distribution of the weighted voting system [5: 3, 2, 1]. Compare your answers in (a) and (b).

(a) $\{P_1, P_2\}$ $\{P_1, P_2, P_3\}$

↳ B_1: 11 B_2: 11 B_3: /4 T

$\beta_1 = 1/2$ $\beta_2 = 1/2$ $\beta_3 = 0$

↳ reduce

(b) $\{P_1, P_2\}$ $\{P_1, P_2, P_3\}$

↳ B_1: 11 B_2: 11 B_3: /4 T

$\beta_1 = 1/2$ $\beta_2 = 1/2$ $\beta_3 = 0$

17. Consider the weighted voting system $[q: 8, 4, 2, 1]$. Find the Banzhaf power distribution of this weighted voting system when

(a) $q = 8$

(b) $q = 9$

(c) $q = 10$

(d) $q = 12$

(e) $q = 14$

(a) $\{P_1\}$ $\{P_1 P_2\}$ $\{P_1 P_3\}$ $\{P_1 P_4\}$
$\{P_1 P_2 P_3\}$ $\{P_1 P_2 P_4\}$ $\{P_1 P_2 P_3 P_4\}$

↳ $B_1 = 7$ $B_2 = 0$ $B_3 = 0$ $B_4 = 0$

$T = 7$

$\mathcal{B}_1 = 1$ $\mathcal{B}_2 = \mathcal{B}_3 = \mathcal{B}_4$

(b) $[9 : 8, 4, 2, 1]$ $\{P_1 P_2\}$ $\{P_1 P_3\}$ $\{P_1 P_4\}$
$\{P_1 P_2 P_3\}$ $\{P_1 P_2 P_4\}$ $\{P_1 P_3 P_4\}$
$\{P_1 P_2 P_3 P_4\}$

↳ $B_1 = 7$ $B_2 = 1$ $B_3 = 1$ $B_4 = 1$

$T = 7+1+1+1 = 10$ //

$\mathcal{B}_1 = 7/10$ $\mathcal{B}_2 = 1/10$ $\mathcal{B}_3 = 1/10$ $\mathcal{B}_4 = 1/10$

(c) $[10 : 8, 4, 2, 1]$ $\{P_1 P_2\}$ $\{P_1 P_3\}$
$\{P_1 P_2 P_3\}$ $\{P_1 P_2 P_4\}$ $\{P_1 P_3 P_4\}$
$\{P_1 P_2 P_3 P_4\}$

↳ $B_1 = 6$ $B_2 = 2$ $B_3 = 2$ $B_4 = 0$

$T = 6+2+2+0 = 10$ //

$\mathcal{B}_1 = 6/10 = 3/5$ $\mathcal{B}_2 = 2/10 = 1/5$ $\mathcal{B}_3 = 2/10 = 1/5$ $\mathcal{B} = 0$

(d) $[12 : 8, 4, 2, 1]$ $\{P_1 P_2\}$ $\{P_1 P_2 P_3\}$ $\{P_1 P_2 P_4\}$ $\{P_1 P_2 P_3 P_4\}$

↳ $B_1 = 4$ $B_2 = 4$ $B_3 = 0$ $B_4 = 0$

$T = 4+4+0+0 = 8$ // $\mathcal{B}_1 = 4/8 = 1/2$ $\mathcal{B}_2 = 4/8 = 1/2$ $\mathcal{B}_3 = \mathcal{B}_4 = 0$

(e) $[14 : 8, 4, 2, 1]$ $\{P_1 P_2 P_3\}$ $\{P_1 P_2 P_3 P_4\}$

↳ $B_1 = 2$ $B_2 = 2$ $B_3 = 2$ $B_4 = 0$ $T = 6$

$\mathcal{B}_1 = 1/3$ $\mathcal{B}_2 = 1/3$ $\mathcal{B}_3 = 1/3$ $\mathcal{B}_4 = 0$