EXERCISES

WALKING

A Weighted Voting Systems

1. Consider the weighted voting system [65: 30, 28, 22, 15, 7, 6]. [*Historical footnote*: This weighted voting system describes the voting in the Nassau County Board of Supervisors (N.Y.) in the 1990s.] Find

   (a) the total number of players.

   (b) the total number of votes.

   (c) the weight of P_2.

   (d) the minimum percentage of the votes needed to pass a motion (rounded to the next whole percent).

→ [65: 30, 28, 22, 15, 7, 6] (P_2)

6 weights mean 6 players

(a) 6 players

(b) 108

(c) 28

(d) 61%

```
  28
  30
+ 28
  22
  15
   7
   6
 ---
 108
```

$\frac{65}{108} = .6018\ldots$ round up

61%

2. Consider the weighted voting system [62: 10, 10, 10, 10, 8, 5, 5, 5, 5, 4, 4, 3, 3, 3, 2]. [*Historical footnote*: This weighted voting system describes the voting in the European Union Council of Ministers prior to 2004.) Find

   (a) the total number of players.

   (b) the total number of votes.

   (c) the weight of P_6.

   (d) the minimum percentage of the votes needed to pass a motion (rounded to the next whole percent).

(a) 15

(b) 87

(c) 5

(d) 72%

$\frac{62}{87} = .7126$

72%

3. Consider the weighted voting system $[q: 10, 6, 5, 4, 2]$.

(a) What is the smallest value that the quota q can take?

(b) What is the largest value that the quota q can take?

(c) What is the value of the quota if *at least* two-thirds of the votes are required to pass a motion?

(d) What is the value of the quota if *more* than two-thirds of the votes are required to pass a motion?

→ [q : 10, 6, 5, 4, 2]

(a) 10+6+5+4+2 = 27

27/2 → 13.5 → round up → 14

(a) 14 ← ½ the votes, then rounded up

(b) 27 ← all votes

(c) 18 ← "at least": don't round up

(d) 19 ← it's 2/3's plus one.

(c) $\frac{2}{3} \cdot 27 = \frac{2}{1} \cdot \frac{27^{9}}{1} = 18$

4. Consider the weighted voting system $[q: 6, 4, 3, 3, 2, 2]$.

(a) What is the smallest value that the quota q can take?

(b) What is the largest value that the quota q can take?

(c) What is the value of the quota if *at least* three-fourths of the votes are required to pass a motion?

(d) What is the value of the quota if *more* than three-fourths of the votes are required to pass a motion?

→ [q : 6, 4, 3, 3, 2, 2]

(a) 6+4+3+3+2+2 = 20/2 = 10 → 11 → round up 1

(c) $\frac{3}{4} \cdot \frac{20^{5}}{1} = 15$

(a) 11

(b) 20

(c) 15

(d) 16 ← plus one

5. A committee has four members (P_1, P_2, P_3 and P_4). In this committee P_1 has twice as many votes as P_2; P_2 has twice as many votes as P_3; P_3 and P_4 have the same number of votes. The quota is $q = 49$. For each of the given definitions of the *quota*, describe the committee using the notation $[q: w_1, w_2, w_3, w_4]$. (*Hint*: Write the weighted voting system as $[49: 4x, 2x, x, x]$, and then solve for x.)

(a) The quota is defined as a *simple majority* of the votes.

(b) The quota is defined as *more than two-thirds* of the votes.

(c) The quota is defined as *more than three-fourths* of the votes.

→ $[49: 4x, 2x, x, x]$

(a) "simple majority" means just over ½. so, 49-1 = 48 is ½. 48 × 2 = 96 is total.

$T \cdot \frac{1}{2} = 48$

$4x + 2x + x + x =$

$\frac{8x}{8} = \frac{96}{8} \rightarrow x = 12$

(a) $[49: 48, 24, 12, 12]$

(b) "more than two-thirds"

→ 48 $\left(\frac{3}{2}\right) T \cdot \frac{2}{3} = 48 \left(\frac{3}{2}\right)$

$T = \frac{24}{1} \cdot \frac{3}{1}$

$T = 72$

(b) $[49: 36, 18, 9, 9]$

↳ $\frac{8x}{8} = \frac{72}{8} \rightarrow x = 9$

(c) "more than 3/4s" of votes

→ 48 $\left(\frac{4}{3}\right) T \cdot \frac{3}{4} = 48 \left(\frac{4}{3}\right)$

$T = \frac{16}{1} \cdot \frac{4}{1} =$

$T = 64$

(c) $[49: 32, 16, 8, 8]$

↳ $\frac{8x}{8} = \frac{64}{8} \rightarrow x = 8$

6. A committee has six members (P_1, P_2, P_3, P_4, P_5, and P_6). In this committee P_1 has twice as many votes as P_2; P_2 and P_3 each has twice as many votes as P_4; P_4 has twice as many votes as P_5; P_5 and P_6 have the same number of votes. The quota is $q = 121$. For each of the given definitions of the *quota*, describe the committee using the notation $[q: w_1, w_2, w_3, w_4, w_5, w_6]$. (*Hint*: Write the weighted voting system as $[121: 8x, 4x, 4x, 2x, x, x]$, and then solve for x.)

(a) The quota is defined as a *simple majority* of the votes.

(b) The quota is defined as *more than two-thirds* of the votes.

(c) The quota is defined as *more than three-fourths* of the votes.

$[121: 8x, 4x, 4x, 2x, x, x]$

(a) "simple majority"

↳ 120 means total is 240.

$8x + 4x + 4x + 2x + x + x = 240$

$\frac{20x}{20} = \frac{240}{20}$

$x = 12$

(a) $[121: 96, 48, 48, 24, 12, 12]$

(b) "more than 2/3 s."

↳ 120. $\left(\frac{3}{3}\right)T \cdot \frac{2}{3} = 120\left(\frac{3}{3}\right)$

$T = 180$

↳ $\frac{20x}{20} = \frac{180}{20} \rightarrow x = 9$

(b) $[121: 72, 36, 36, 18, 9, 9]$

(c) "more than 3/4s"

↳ 120. $\left(\frac{4}{4}\right)T \cdot \frac{3}{4} = 120\left(\frac{4}{4}\right)$

$T = 160.$

↳ $\frac{20x}{20} = \frac{160}{20} \rightarrow x = 8$

(c) $[121: 64, 32, 32, 16, 8, 8]$

7. In each of the following weighted voting systems, determine which players, if any, (i) are dictators; (ii) have veto power; (iii) are dummies.

(a) [15: 16, 8, 4, 1]

(b) [18: 16, 8, 4, 1]

(c) [24: 16, 8, 4, 1]

(a) P_1 = dictator. rest are dummies

(b) P_1 = veto power. P_4 is a dummy. votes is 29. 29 − 16 = 13, which is not enough to pass. $q = 18$.

(c) $P_1 + P_2$ = veto power. $P_3 + P_4$ are dummies.

↳ if P_1 (16) or P_2 (8) vote no, it won't pass with $q = 24$.

8. In each of the following weighted voting systems, determine which players, if any, (i) are dictators; (ii) have veto power; (iii) are dummies.

(a) [8: 8, 4, 2, 1]

(b) [9: 8, 4, 2, 1]

(c) [12: 8, 4, 2, 1]

(d) [15: 8, 4, 2, 1]

(a) P_1 is a dictator. rest are dummies

(b) P_1 has veto power.

(c) $P_1 + P_2$ have veto power. rest are dummies

↳ w/o P_1 or P_2, no thing can pass with $q = 12$.

(d) All players have veto power.

9. Consider the weighted voting system $[q: 8, 4, 2]$. Find the *smallest* value of q for which

(a) all three players have veto power.

(b) P_2 has veto power but P_3 does not.

(c) P_3 is the only dummy.

(a) 13

(b) 11

(c) 11

(a) $8+4=12$, must be bigger than this.... 13.

(b) $8+2=10$ .... bigger → 11

no P_2 ↑

(c) if its Total $- P_3$ =

↓

$14 - 2 = 12$

can it be smaller? yes, 11.

can it be smaller? no, because if its 10, then P_3 has power, cuz $8+2=10$.

10. Consider the weighted voting system $[q: 7, 5, 3]$. Find the *smallest* value of q for which

(a) all three players have veto power.

(b) P_2 has veto power but P_3 does not.

(c) P_3 is the only dummy.

(a) 13

(b) 11

(c) 11