Discrete N: D: P: 1 2 3 4 5 6

Topic 01 // Lesson 05 (notes)

Rankings

Extended Ranking Methods

Ex 1.17 Math Election // ► **Extended Plurality**

# of voters	14	10	8	4	1
1st	A	C	D	B	C
2nd	B	B	C	D	D
3rd	C	D	B	C	B
4th	D	A	A	A	A

A =

B =

C =

D =

1st =

2nd =

3rd =

4th =

Ex 1.18 Math Election

► **Extended Borda Count**

A = 79 points

B = 106 points

C = 104 points

D = 81 points

1st =

2nd =

3rd =

4th =

Ex 1.19 Math Election

► **Extended Plurality w/ Elimination**

1st to get eliminated was B, so last place

Next to be eliminated was C, so 2nd to last….

1st =

2nd =

3rd =

4th =

Ex 1.20 Math Election

► **Extended Pairwise Comparisons**

A = 0 pts

B = 2 pts

C = 3 pts

D = 1 pt

1st =

2nd =

3rd =

4th =

▼ **Table of Four Methods**

	1st	**2nd**	**3rd**	**4th**
Extended Plurality				
Extended Borda Count				
Extended Plurality w/ Elimination				
Extended Pairwise Comparisons				

Note:

Recursive Ranking Methods

→

Ex 1.21 Math Election // ► **Recursive Plurality**

# of voters	**14**	**10**	**8**	**4**	**1**
1st	A	C	D	B	C
2nd	B	B	C	D	D
3rd	C	D	B	C	B
4th	D	A	A	A	A

→

# of voters	**14**	**10**	**8**	**4**	**1**
1st					
2nd					
3rd					

→

# of voters	**14**	**10**	**8**	**4**	**1**
1st					
2nd					

1st	2nd	3rd	4th

Ex 1.22 Math Election // ► **Recursive Plurality w/ Elimination**

# of voters	**14**	**10**	**8**	**4**	**1**
1st	A	C	D	B	C
2nd	B	B	C	D	D
3rd	C	D	B	C	B
4th	D	A	A	A	A

→

# of voters	**14**	**10**	**8**	**4**	**1**
1st					
2nd					
3rd					

→

# of voters	**14**	**10**	**8**	**4**	**1**
1st					
2nd					

1st	2nd	3rd	4th

Conclusion:

Conclusion

1.

2.

3.

4.

Majority Criterion:

Condorcet Criterion:

Monotonicity Criterion:

Independence of Irrelevant Alternatives (IIA):

►

► Plurality violates Condorcet and IIA Criterions. Borda Count violates the Majority, Condorcet and IIA Criterions. Plurality w/ Elimination violates the Condorcet, Monotonocity, and IIA Criterion. Pairwise Comparisons violates the IIA Criterion

Arrow's Impossibility Theorem

Discrete

N: D: P: 1 2 3 4 5 6

Topic 01 // Lesson 05 (notes)

Rankings

For not just first place, but list all places

Extended Ranking Methods

Ex 1.17 Math Election // ► **Extended Plurality**

# of voters	14	10	8	4	1
1st	A	C	D	B	C
2nd	B	B	C	D	D
3rd	C	D	B	C	B
4th	D	A	A	A	A

A = 14

B = 4

C = 11

D = 8

1st = A

2nd = C

3rd = D

4th = B

Ex 1.18 Math Election

► **Extended Borda Count**

A = 79 points

B = 106 points

C = 104 points

D = 81 points

1st = B

2nd = C

3rd = D

4th = A

Ex 1.19 Math Election

► **Extended Plurality w/ Elimination**

1st to get eliminated was B, so last place

Next to be eliminated was C, so 2nd to last….

1st = D

2nd = A

3rd = C

4th = B

Ex 1.20 Math Election

► **Extended Pairwise Comparisons**

A = 0 pts

B = 2 pts

C = 3 pts

D = 1 pt

1st = C

2nd = B

3rd = D

4th = A

▼ **Table of Four Methods**

	1st	2nd	3rd	4th
Extended Plurality	A	C	D	B
Extended Borda Count	B	C	D	A
Extended Plurality w/ Elimination	D	A	C	B
Extended Pairwise Comparisons	C	B	D	A

Note: the wide differences through the methods. This is to show that things can get very crazy in elections, but there tend to be more consistency in real-life

Recursive Ranking Methods

→ Similar to a "feedback loop". Rank, Remove, Repeat.

Ex 1.21 Math Election // ► **Recursive Plurality**

# of voters	14	10	8	4	1
1st	A	C	D	B	C
2nd	B	B	C	D	D
3rd	C	D	B	C	B
4th	D	A	A	A	A

→

# of voters	14	10	8	4	1
1st	B	C	D	B	C
2nd	C	B	C	D	D
3rd	D	D	B	C	B

→

# of voters	14	10	8	4	1
1st	C	C	D	D	C
2nd	D	D	C	C	D

1st A	2nd B	3rd C	4th D

Ex 1.22 Math Election // ► **Recursive Plurality w/ Elimination**

# of voters	14	10	8	4	1
1st	A	C	D	B	C
2nd	B	B	C	D	D
3rd	C	D	B	C	B
4th	D	A	A	A	A

→

# of voters	14	10	8	4	1
1st	A	C	C	B	C
2nd	B	B	B	C	B
3rd	C	A	A	A	A

→

# of voters	14	10	8	4	1
1st	A	B	B	B	B
2nd	B	A	A	A	A

1st D	2nd C	3rd B	4th A

Conclusion:

Recursive and Extended Methods can produce very different results. There is no simple answer of which is better. In real-life elections extended methods are almost always used.

Conclusion

1. Elections are more than just for president and governors. It is for deciding where to eat, getting a job, etc.
2. There are many different methods to vote
3. Outcomes can change with different voting strategies
4. Elections should be fair.

Majority Criterion: A majority candidate should always win the election

Condorcet Criterion: A Condorcet Candidate (winner of head-to-head) should always win the election.

Monotonicity Criterion: If candidate X wins an election, then a second election where that candidate gains votes should still win the election.

Independence of Irrelevant Alternatives (IIA): If candidate X wins an election, then a second election where a candidate exits or irrelevant candidates enters the race, then Candidate X should still win.

► A fair voting method should clearly satisfy all four of the above

► Plurality violates Condorcet and IIA Criterions. Borda Count violates the Majority, Condorcet and IIA Criterions. Plurality w/ Elimination violates the Condorcet, Monotonocity, and IIA Criterion. Pairwise Comparisons violates the IIA Criterion

Arrow's Impossibility Theorem

It is mathematically impossible for a democratic voting method to satisfy all of the fairness criteria.