Discrete N: D: P: 1 2 3 4 5 6

Topic 01 // Lesson 04 (notes)

Method of Pairwise Comparisons

Like a Round Robin Tournament in which every player plays once against every other, where *head-to-head* matches are called **pairwise comparisons**.

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Ex 1.11

# of voters	14	10	8	4	1
1st	A	C	D	B	C
2nd	B	B	C	D	D
3rd	C	D	B	C	B
4th	D	A	A	A	A

A vs B: B vs C:

A vs C: B vs D:

A vs D: C vs D:

Overall Tally: A = B= C= D=

The Winner is:

Ex 1.12 LAXer's Draft Choice Election

# of voters	**2**	**6**	**4**	**1**	**1**	**4**	**4**
1st	A	B	B	C	C	D	E
2nd	D	A	A	B	D	A	C
3rd	C	C	D	A	A	E	D
4th	B	D	E	D	B	C	B
5th	E	E	C	E	E	B	A

Overall Tally: A = B= C= D= E=

The Winner is:

Trial 2: Without Player C

# of voters	**2**	**6**	**4**	**1**	**1**	**4**	**4**
1st							
2nd							
3rd							
4th							

Overall Tally: A = B= D= E=

The Winner is:

Independence of Irrelevant Alternatives Criterion (IIA)

Alternative Interpretation: If Candidate X is a winner of an election and in a reelection another candidate that has no chance of winning (an "irrelevant alternative") enters the race, then X should still be the winner

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↓ How many Pairwise Comparisons?

→ Long Sums

Add up 1 to 99.

Gauss:

Sum of Consecutive Integers Formula

→ Counting Pairwise Comparisons

Consider election with 10 candidates: A, B, C, D, E, F, G, H, I, and J

Compare A with 9 others is

Compare B with 8 others is

Compare C with 7 others is

Compare D with 6 others is

Continuing until J…

Total #:

Number of Pairwise Comparisons

In an election with N candidates the total number of pairwise comparisons is

Practice 1. How many comparisons would occur in a 12 candidate race?

Practice 2. How many comparisons would occur in a 20 candidate race?

Discrete N: D: P: 1 2 3 4 5 6

Topic 01 // Lesson 04 (notes)

Method of Pairwise Comparisons

Like a Round Robin Tournament in which every player plays once against every other, where *head-to-head* matches are called **pairwise comparisons**.

- Each candidate goes head-to-head and winner gets a single point
- If a tie, then both get 1/2 point
- Winner is candidate with most points
- Ties are very common in this method

Ex 1.11

# of voters	14	10	8	4	1
1st	A	C	D	B	C
2nd	B	B	C	D	D
3rd	C	D	B	C	B
4th	D	A	A	A	A

A vs B: 14 to 23, B=1

B vs C: 18 to 19, C=1

A vs C: 14 to 23, C=1

B vs D: 28 to 9, B=1

A vs D: 14 to 23, D=1

C vs D: 25 to 12, C=1

Overall Tally: A = 0 B= 2 C= 3 D= 1

The Winner is: Person C

Ex 1.12 LAXer's Draft Choice Election

# of voters	**2**	**6**	**4**	**1**	**1**	**4**	**4**
1st	A	B	B	C	C	D	E
2nd	D	A	A	B	D	A	C
3rd	C	C	D	A	A	E	D
4th	B	D	E	D	B	C	B
5th	E	E	C	E	E	B	A

A vs B: B = 1

B vs E: B= 1

A vs C: A= 1

C vs D: C = 1

A vs D: A= 1

C vs E: E = 1

A vs E: A= 1

D vs E: D = 1

B vs C: C = 1

B vs D: B=$^1/_2$; D=$^1/_2$;

Overall Tally: A = 3 B= 2 $^1/_2$ C= 2 D= 1 $^1/_2$ E= 1

The Winner is: Person A

Trial 2: Without Player C

# of voters	**2**	**6**	**4**	**1**	**1**	**4**	**4**
1st	A	B	B	B	D	D	E
2nd	D	A	A	A	A	A	D
3rd	B	D	D	D	B	E	B
4th	E	E	E	E	E	B	A

A vs B: B = 1

B vs E: B= 1

A vs D: A= 1

D vs E: D = 1

A vs E: A= 1

B vs D: B=$^1/_2$; D=$^1/_2$;

Overall Tally: A = 2 B= 2 $^1/_2$ D= 1 $^1/_2$ E= 0

The Winner is: Person B

Independence of Irrelevant Alternatives Criterion (IIA)

If Candidate X is a winner of an election and in a recount one of the *nonwinning* candidates withdraws or is disqualified, then X should still be a winner of the election.

Alternative Interpretation: If Candidate X is a winner of an election and in a reelection another candidate that has no chance of winning (an "irrelevant alternative") enters the race, then X should still be the winner

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↓ How many Pairwise Comparisons?

→ Long Sums

Add up 1 to 99.

Gauss:

Sum of Consecutive Integers Formula

$$1 + 2 + 3 + \ldots + L = \frac{L(L+1)}{2}$$

→ Counting Pairwise Comparisons

Consider election with 10 candidates: A, B, C, D, E, F, G, H, I, and J

Compare A with 9 others is 9 comparisons

Compare B with 8 others is 8 comparisons

Compare C with 7 others is 7 comparisons

Compare D with 6 others is 6 comparisons

Continuing until J…

Total #: 1 + 2 + 3 + ..+ 8 + 9 =

$$\frac{9 \times 10}{2} = 45$$

Number of Pairwise Comparisons

In an election with N candidates the total number of pairwise comparisons is $\frac{(N-1)N}{2}$

Practice 1. How many comparisons would occur in a 12 candidate race?

Practice 2. How many comparisons would occur in a 20 candidate race?