

Find the least common denominator (LCD) of the following rational expressions.

1) $\frac{1}{x^2-81}$, $\frac{5}{x^2-18x+81}$, and $\frac{4}{x^3+9x^2}$

$(x+9)(x-9)$
 $(x-9)(x-9)$
 $x^2(x+9)$

$x^2(x-9)^2(x+9)$

2) $\frac{3}{x^2-9}$, $\frac{7}{x^2+3x}$, and $\frac{2}{x^3}$

$x(x+3)$
 $(x+3)(x-3)$

$x^3(x+3)(x-3)$

Simplify each expression.

3) $\frac{3x+12}{x^2-16}$

$= \frac{3(x+4)}{(x+4)(x-4)}$

$= \frac{3}{x-4}$

4) $\frac{x^3-x}{x^2+5x-6} = \frac{x(x^2-1)}{(x+6)(x-1)}$

$= \frac{x(x+1)(x-1)}{(x+6)(x-1)}$

$\frac{x(x+1)}{x+6}$

5) $\frac{x^2+5x-14}{x^2-49}$

$= \frac{(x+7)(x-2)}{(x+7)(x-7)}$

$\frac{x-2}{x-7}$

6) $\frac{8x^2-8}{1-x} = \frac{8(x^2-1)}{-1(x-1)}$

$= \frac{8(x+1)(x-1)}{-1(x-1)}$

$\frac{-8(x+1)}{or -8x-8}$

7) $\frac{3x^2-3x-60}{5-x} = \frac{3(x^2-x-20)}{-1(x-5)}$

$= \frac{3(x-5)(x+4)}{-1(x-5)}$

$-3(x+4)$

8) $\frac{2x^3y+20x^2y+50xy}{2xy}$

$= \frac{2xy(x^2+10x+25)}{2xy}$

$= (x+5)(x+5)$

$(x+5)^2$

9) $\frac{3x^3y^2+24x^2y^3+48xy^4}{3xy^2}$

$= \frac{3xy^2(x^2+8xy+16y^2)}{3xy^2}$

$= (x+4y)(x+4y)$

$(x+4y)^2$

Perform the indicated operation and simplify.

10) $\frac{(x+3)^2}{x^2-2x-15} \cdot \frac{x^2-5x}{x^2-9}$

$$\frac{\cancel{(x+3)}\cancel{(x+3)}}{\cancel{(x-5)}\cancel{(x+3)}} \cdot \frac{x\cancel{(x-5)}}{\cancel{(x+3)}(x-3)} = \frac{\cancel{(x+3)}\cancel{(x+3)} \cdot x\cancel{(x-5)}}{\cancel{(x-5)}\cancel{(x+3)} \cdot \cancel{(x+3)}(x-3)}$$

$$\boxed{\frac{x}{x-3}}$$

11) $\frac{x^2+6x+9}{x^2-3x-10} \cdot \frac{3x-15}{x^2-9}$

$$\frac{\cancel{(x+3)}\cancel{(x+3)}}{\cancel{(x-5)}(x+2)} \cdot \frac{3\cancel{(x-5)}}{\cancel{(x+3)}(x-3)}$$

$$\boxed{\frac{3(x+3)}{(x+2)(x-3)}}$$

12) $\frac{x^2+3x-40}{2x^2+8x} \div \frac{x^2-64}{4x+16}$ ← flip

$$\frac{x^2+3x-40}{2x^2+8x} \cdot \frac{4x+16}{x^2-64}$$

$$\frac{\cancel{(x+8)}\cancel{(x-5)}}{2x\cancel{(x+4)}} \cdot \frac{4\cancel{(x+4)}}{\cancel{(x+8)}(x-8)}$$

$$\boxed{\frac{2(x-5)}{x(x-8)}}$$

13) $\frac{2x^2-72}{4x^2-24x} \div \frac{x^2+3x-18}{1}$ flip

$$\frac{2x^2-72}{4x^2-24x} \cdot \frac{1}{x^2+3x-18}$$

$$\frac{2(x^2-36)}{4x(x-6)} \cdot \frac{1}{(x+6)(x-3)}$$

$$\frac{2\cancel{(x+6)}\cancel{(x-6)}}{4x\cancel{(x-6)}} \cdot \frac{1}{\cancel{(x+6)}(x-3)}$$

$$\boxed{\frac{1}{2x(x-3)}}$$

14) $\frac{3x-30}{1} \div \frac{4x^2-400}{x^2+15x+50}$ flip

$$\frac{3x-30}{1} \cdot \frac{x^2+15x+50}{4(x^2-100)}$$

$$\frac{3\cancel{(x-10)}}{1} \cdot \frac{\cancel{(x+10)}(x+5)}{4\cancel{(x+10)}\cancel{(x-10)}}$$

$$\boxed{\frac{3(x+5)}{4}}$$

15) $\frac{3x+1}{x^2-5x} + \frac{7 \cdot x}{x-5 \cdot x}$

$$\frac{3x+1}{x(x-5)} + \frac{7x}{x(x-5)}$$

$$\frac{3x+1+7x}{x(x-5)}$$

$$\boxed{\frac{10x+1}{x(x-5)}}$$

16) $\frac{2x}{x^2-6x-16} + \frac{5(x-8)}{x+2(x-8)}$

$$\frac{2x}{(x-8)(x+2)} + \frac{5x-40}{(x+2)(x-8)}$$

$$\boxed{\frac{7x-40}{(x+2)(x-8)}}$$

17) $\frac{4}{x^2-5x-24} + \frac{x-2}{x+3} \frac{(x-8)}{(x-8)}$

$$\frac{4}{(x-8)(x+3)} + \frac{(x-2)(x-8)}{(x-8)(x+3)}$$

$$\frac{4 + (x-2)(x-8)}{(x-8)(x+3)}$$

$$\frac{4 + x^2 - 8x - 2x + 16}{(x-8)(x+3)}$$

$$\boxed{\frac{x^2-10x+20}{(x-8)(x+3)}}$$

18) $\frac{(x-5)^6}{x+3} - \frac{2}{x-5} \frac{(x+3)}{(x+3)}$

$$\frac{6(x-5)}{(x-5)(x+3)} - \frac{2(x+3)}{(x+3)(x-5)}$$

$$\frac{6x-30-2x-6}{(x-5)(x+3)}$$

$$\boxed{\frac{4x-36}{(x-5)(x+3)}}$$

or $\frac{4(x-9)}{(x-5)(x+3)}$

$$24) \frac{x^2}{x^2} + \frac{1}{x^2} = \frac{3 \cdot x}{x \cdot x} \quad \text{top}$$

$$\hookrightarrow x^2 \quad x^2 + 1 = 3x$$

$$\underline{-3x} \quad \underline{-3x}$$

$$x^2 - 3x + 1 = 0$$

a b c

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$

$$\boxed{\frac{3 \pm \sqrt{5}}{2}}$$

$$25) \frac{x^2}{x^2} + \frac{1}{x^2} = \frac{7 \cdot x}{x \cdot x}$$

$$\hookrightarrow x^2 \quad 2x^2 + 1 = 7x$$

$$\underline{-7x} \quad \underline{-7x}$$

$$2x^2 - 7x + 1 = 0$$

a b c

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{7 \pm \sqrt{49-8}}{4} = \frac{7 \pm \sqrt{41}}{4}$$

$$\boxed{\frac{7 \pm \sqrt{41}}{4}}$$

$$26) \frac{x^2}{x^2} + \frac{2}{x^2} = \frac{5 \cdot x}{x \cdot x}$$

$$\hookrightarrow x^2 \quad x^2 + 2 = 5x$$

$$\underline{-5x} \quad \underline{-5x}$$

$$x^2 - 5x + 2 = 0$$

a b c

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{25-8}}{2} = \frac{5 \pm \sqrt{17}}{2}$$

$$\boxed{\frac{5 \pm \sqrt{17}}{2}}$$

Problems 27-29: If x is a real number, for what values of x is each equation true? Choose from

- A) All values of x
- B) No values of x
- C) Some values of x
- D) Impossible to determine

27) $\frac{3x+18}{3} = x+6$

28) $\frac{6x-2}{2} = 3x-2$

29) $\frac{x^2-36}{x+6} = x-6$

$$\frac{3(x+6)}{3} = x+6$$

A All

$$\frac{2(3x-1)}{2} = 3x-2$$

B None

$$\frac{(x+6)(x-6)}{x+6} = x-6$$

$$x+6=0$$

$$-6 = -6$$

$$x = -6$$

C

because it doesn't work for -6!

Problems 30-32: Which of the following statements is correct about each equation.

- A) The equation is always true.
- B) The equation is always true, except when $x = -7$.
- C) The equation is never true.
- D) The equation is sometimes true when $x = -7$.

30) $\frac{4x-28}{4} = x-28$

31) $\frac{2x+14}{2} = x+7$

32) $\frac{x^2-49}{x+7} = x-7$

$$\frac{4(x-7)}{4} = x-28$$

C

$$\frac{2(x+7)}{2} = x+7$$

A

$$\frac{(x+7)(x-7)}{x+7} = x-7$$

B