Algebra 2 // BM 4 Review A

1) Solve for x. $|5-10x| = 25$

$5-10x = 25$ or $5-10x = -25$

$-5 \quad -5$ $\quad$ $-5 \quad -5$

$\frac{-10x}{-10} = \frac{20}{-10}$ $\quad$ $\frac{-10x}{-10} = \frac{-30}{-10}$

$x = -2$ $\quad$ $x = 3$

▶ Absolute Values

After $|\ |=\#$, break into 2 problems.

2) ① $2x + 3y + z = 5$

② $3x + 3y - z = -1$

③ $3x + 2y - z = 0$

① $2x + 3y + z = 5$
② $3x + 3y - z = -1$

④ $5x + 6y = 4$

① $2x + 3y + z = 5$
③ $3x + 2y - z = 0$

⑤ $5x + 5y = 5$

$5x + 6y = 4$

$(-1)\ 5x + 5y = 5 \rightarrow -5x - 5y = -5$

$y = -1$

④ $5x + 6y = 4$

$5x + 6(-1) = 4$

$5x - 6 = 4$

$+6 \quad +6$

$\frac{5x}{5} = \frac{10}{5}$

$x = 2$

① $2x + 3y + z = 5$

$2(2) + 3(-1) + z = 5$

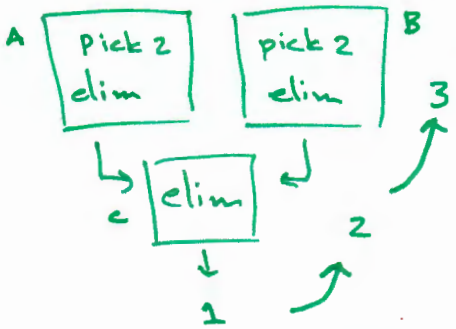
$4 - 3 + z = 5$

$1 + z = 5$

$-1 \quad -1$

$z = 4$

▶ 3 Variable



↑ Make sure to cancel the same variable in A and B.

$(2, -1, 4)$

3) $a + b = 10$ ← represents "packages"

$6a + 8b = 68$ ← represents "individual bagels"

▶ the key idea is to keep similar things together.

↳ $-6(a+b)=10 \rightarrow -6a - 6b = -60$

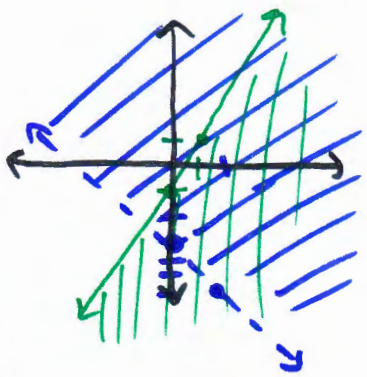
$6a + 8b = 68$

$$\frac{2b}{2} = \frac{8}{2} \rightarrow b = 4$$

4 packages of 8-bagels.

4) Solve system of inequalities

$$\begin{cases} 3x + 2y > -6 \\ y \leq 2x - 1 \checkmark \end{cases}$$



$3x + 2y > -6$

$-3x \quad -3x$

$\frac{2y}{2} > \frac{-3x}{2} - \frac{6}{2}$

$y > -\frac{3}{2}x - 3 \checkmark$

▶ First, equations must look like $y \gtrless mx + b$

Second, graph

Third, shade

↳ Overlap or Cross over is where these equations both work.

5) Find $(8i)(5i) = 40i^2$ ← $i^2 = -1$

$= 40(-1) = \boxed{-40}$

6) Product of $(4+i)$ and $(4-i)$?

↓ means multiply

$(4+i)(4-i) = 16 - 4i + 4i - i^2$

$= 16 - i^2$

$= 16 - (-1) =$

$= 16 + 1 = \boxed{17}$

(7) Value of i^6?

$\boxed{-1}$

▶ $i^1 = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

↑ just cycle through. <Count>

← Remember: I WON I WON! or I ONE I ONE then negatives in the middle.

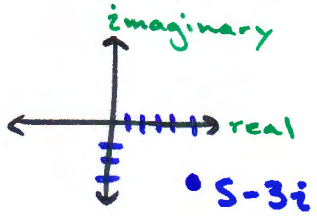
(8) What is $\frac{2}{4+i}$?

$$\frac{2}{4+i} \cdot \frac{4-i}{4-i} = \frac{8-2i}{17} = \boxed{\frac{8}{17} - \frac{2}{17}i}$$

↳ $16 - 4i + 4i - i^2 = 16 - i^2 = 16 - (-1) = 17$ ✓

← Same as you did in #6.

(9) Sketch $5-3i$



Think → $5-3i$ → x y → $(5,-3)$ ↩

(10) Solutions to: $x^2 + 2x + 10 = 0$

↳ have to use quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a=1$ $b=2$ $c=10$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(10)}}{2(1)} = \frac{-2 \pm \sqrt{4 - 4(10)}}{2} = \frac{-2 \pm \sqrt{-36}}{2}$$

↖ NO! it's a negative... ...so imagine that you can do it!

$$= \frac{-2 \pm \sqrt{36}\,i}{2} = \frac{-2 \pm 6i}{2} = \frac{-2}{2} \pm \frac{6}{2}i$$

$$= \boxed{-1 \pm 3i}$$

▶ Solutions, roots, zeros, and x-intercepts are all the same. Use: x-factor or quadratic formula

(11) x-intercepts of: $8x^2-2x-1=0$

1·8, 2·4

4		1
2	-4	-1
8	-2	-1

1·-1 → $(4x+1)$

→ $(2x-1)$

$4x+1=0$ and $2x-1=0$

$-1 \quad -1$ $\quad +1 \quad +1$

$\frac{4x}{4} = \frac{-1}{4}$ $\quad \frac{2x}{2} = \frac{1}{2}$

$x = -1/4$ $\quad x = 1/2$

$(-1/4, 0)$ $\quad (1/2, 0)$

Note: you may see the answer written as: $-1/4$ and $1/2$

12) vertex of: $y = x^2 - 4x + 5$ $\quad a=1 \; b=-4 \; c=5$ $\quad x = \frac{-b}{2a}$

$x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$

↳ plug back in for y.

(x, y) or (h, k)

$y = x^2 - 4x + 5$

$y = (2)^2 - 4(2) + 5 = 4 - 8 + 5 = -4 + 5 = 1$

$(2, 1)$

13) $y = x + 3$

$xy = x + y + 17 \rightarrow x(x+3) = x + (x+3) + 17$

$x^2 + 3x = 2x + 20$

$-20 \quad -2x \quad -2x \quad -20$

$x^2 + x - 20 = 0$ ← x-factor of quadratic

1·1

1		5
1		-4

-1·20 $\quad -4+5$

1·-20 $\quad 4-5$

2·-10

-2·10

$(x+5)(x-4)$

$x+5=0$ $\quad y = x+3$ $\quad x-4=0$ $\quad y = x+3$

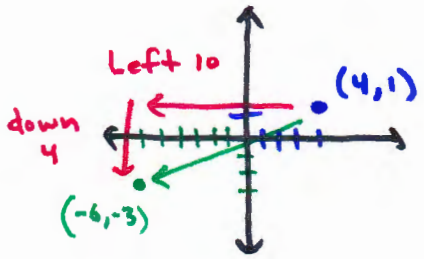
$-5 \quad -5$ $\quad y = (-5)+3$ $\quad +4 \quad +4$ $\quad y = 4+3$

$x = -5$ $\quad y = -2$ $\quad x = 4$ $\quad y = 7$

$(-5, -2)$ $\quad (4, 7)$

14) Describe the translation: $y=(x-4)^2+1$ to $y=(x+6)^2-3$

vertex (4,1) → vertex (-6,-3)



Left 10, Down 4

▶ Be Careful of going backwards. 1st → 2nd

15) Which of the following sentences is true about the graphs of $y=-3(x+4)^2-1$ and $y=2(x+4)^2-1$?

$y=-3(x+4)^2-1$: ↳ vertex is: (-4,-1); ↳ a = -3

$y=2(x+4)^2-1$: a = 2 ↳ ↳ vertex is (-4,-1)

A) Their vertices are minimums.
B) The graphs have the same shape with different vertices. ↘ same vertices
C) The graphs have different shapes with different vertices.
D) One graph has a vertex that is a maximum, while the other graph has a vertex that is a minimum.

↳ the a's are ~~different~~ so they are different shapes.
↳ one a is +, one is −, so one is going up, the other down.
↓ minimum ↓ maximum
→ D ←

16) Cynthia is solving the equation $x^2-6x=5$ by completing the square. What number should be added to both sides of the equation to complete the square?

$x^2-6x+\boxed{9}=5+\boxed{9}$

$\frac{1}{2}$ ↘ ↗ 2

$(x-3)^2$

↑ 9

17) $\dfrac{a^2b^5c^{-3}}{(ab^4c^{-2})^3}=\dfrac{a^2b^5c^{-3}}{a^3b^{12}c^{-6}}=\dfrac{a^2b^5c^6}{a^3b^{12}c^3}=\dfrac{c^3}{ab^7}$

parantheses 1st — move negatives — which has more? by how many?

18) $\frac{30x^{-6}}{7y^3} \div \frac{5x^2}{14y^{-1}} = \frac{30x^{-6}}{7y^3} \cdot \frac{14y^{-1}}{5x^2} = \frac{6\,30}{7y^3x^6} \cdot \frac{14^2}{8x^2y} = \boxed{\frac{12}{x^8y^4}}$

stay ↳ switch ↳ more negatives →

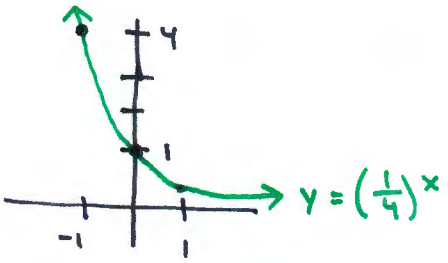
19) If the equation $y = (\frac{1}{4})^x$ is graphed, which of the following values of x would produce a point furthest from the x-axis?

x	y
-1	4
0	1
1	$\frac{1}{4}$

$(\frac{1}{4})^{-1} = 4$

$(\frac{1}{4})^0 = 1$

$(\frac{1}{4})^1 = \frac{1}{4}$



▷ the numbers furthest would be the smallest #, so which is smallest?

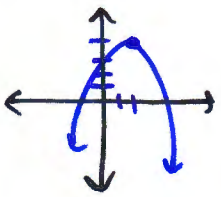
note: numbers closest would be biggest...

A) $\frac{1}{3} \approx .\overline{3}$ B) $\frac{7}{5} \approx 1.4$ C) $\frac{9}{4}$ 2. D) $\frac{11}{2}$

$5\overline{)7.0}$ 1.4

20) Sketch $y = -3(x-2)^2 + 4$ → vertex is (2, 4)

↳ facing down, thin



← Note: very general graph, but enough.

21) Suppose a certain radioactive element decays over time according to the equation $y = A(\frac{1}{2})^{\frac{t}{300}}$, where A = number of grams present initially and t = time in years. If 1600 grams were present initially, how many grams will remain after 900 years?

t = 900 A = 1600

$y = A(\frac{1}{2})^{\frac{t}{300}} = 1600(\frac{1}{2})^{\frac{900}{300}} = 1600(\frac{1}{2})^3 = 1600(\frac{1}{8}) = \frac{1600}{8}$

just plug in → reduce → PEMDAS

$= 200$ grams

A2-12 22) Bacteria in a culture are growing exponentially with time, as shown in the table below.

t y

Bacteria Growth

Day	Bacteria
0	100
1	400
2	1600

↓ let's just plug in and check
if we plug in t=0, then y = 100...
keep checking for t=1, t=2

Which of the following equations expresses the number of bacteria, y, present at any time, t?

~~A) $y = 100+4^t$~~ ← t=0 then y = 101?

~~B) $y = (400)\bullet(2)^t$~~ ← $400\cdot 1$ ← t=0, then y = 400

~~C) $y = (400)\bullet(4)^t$~~ ← t=0 then y = 400

$400\cdot 1 = 400$

D) $y = (100)\bullet(4)^t$ ← t=0, then y = 100

$100\cdot 1$

23) A three-person committee is chosen at random from a group of 8 people. How many different committees are possible?

Committee → Combination

$$_8C_3 = \frac{8!}{3!(8-3)!} = \frac{8!}{3!\,5!} = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{3\cdot 2\cdot 1\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} = \boxed{56}$$

1 5 1

24) A train is made up of a locomotive, 5 different cars, and a caboose. If the locomotive ways can the train be ordered?

1 choice _____ · _____ · _____ · _____ · _____ 1 choice

loco cab

← like Mr Potatohead

$20\cdot 6 = \boxed{120}$

25) A box contains 6 large white marbles, 3 large black marbles, ~~5 small white~~ marbles, and ~~4 small black marbles~~. If a marble is drawn at random, what is the probability that it is white, given that it is one of the big marbles?

white → $\frac{6}{9} = \boxed{\frac{2}{3}}$

total →

↳ means, we don't care about not big, or not large. so ignore the small.

26) Laura and Sam are among 6 students who have qualified for a prize. Two students from the group of six will be selected at random to win prizes. What is the probability that both Laura and Sam will be the 2 students selected?

$$\frac{2}{6} \cdot \frac{1}{5} = \frac{2}{30} = \boxed{\frac{1}{15}}$$

27) How many 5-letter permutations can be made from the letters in the word "**SASSY**"?

$$\frac{5!}{3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = \boxed{20}$$

3 s's →

28) Ben has taken 4 tests in his English class. He has received the following scores. 72, 80, 75, 78

$\bar{x} = 76$

If the mean of these scores is approximately 76, what is the population standard deviation for these scores? (Round the answer to the nearest tenth.)

A) 1.7 B) 3.0 C) 6.1 D) 9.3

σ

	72	80	75	78
$x-\bar{x}$	-4	4	-1	2
$(x-\bar{x})^2$	16	16	1	4

$\rightarrow \frac{16+16+1+4}{4} = \frac{37}{4}$

variance

$\sigma^2 = 9.25$ ←

9.25
$4\overline{)37.00}$
-36
1.0
-8
2.0

So, $\sigma = \sqrt{9.25}$

≈ 3.0

PS-7 29) 4, 2, 3, 1, 3, 5

Bill found the mean and standard deviation of the set of numbers given above. If he adds 3 to each number, which of the following will result?

~~A) The mean will be multiplied by 3.~~
~~B) The standard deviation will increase by 3.~~
~~C) The mean will not change.~~
D) The standard deviation will not change.

let's see. We will need to do this 2x.

4, 2, 3, 1, 3, 5

$\bar{x} = \frac{4+2+3+1+3+5}{6} = \frac{18}{6} = 3$

4	2	3	1	3	5
1	-1	0	-2	0	2
1	1	0	4	0	4

$\sigma^2 = \frac{1+1+0+4+0+4}{6} = \frac{10}{6}$

$\sigma^2 = 1.\overline{6}$

$\sigma = \sqrt{1.\overline{6}}$

7, 5, 6, 4, 6, 8

$\bar{x} = \frac{7+5+6+4+6+8}{6} = \frac{36}{6} = 6$

Notice the mean increased by 3.

7	5	6	4	6	8
1	-1	0	-2	0	2
1	1	0	4	0	4

$\sigma^2 = \frac{1+1+0+4+0+4}{6} = \frac{10}{6}$ ← same

$\sigma^2 = 1.\overline{6}$

$\sigma = \sqrt{1.\overline{6}}$ ← same

$1.\overline{6}$ (6 | 10.0, −6, 40, −36, 4...)

→ so, A is nope. B is nope. The mean increased by 3 (add 3). but, C is nope (because it did change). D! Left and Right is exactly the same!

30) Les wants to create a 5-character password. The first 3 characters must be a letter from his first name, and the last 2 characters must be a digit from the number 9381. How many different passwords are possible? Repetition of letters and digits is **NOT** allowed.

$3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 = 72$

Les 9381

31) A teacher is randomly handing out 8 graphing calculators and 6 scientific calculators. What is the probability that the first calculator he hands out will be a graphing calculator and the second calculator that he hands out will be a scientific calculator?

no replacement.

$$\frac{8}{14} \cdot \frac{6}{13} = \frac{\cancel{8}^{4}}{\cancel{14}_{7}} \cdot \frac{6}{13} = \boxed{\frac{24}{91}}$$

reduces, no replacement.

32) There is a 60% chance that it will rain Friday and a 20% chance that it will rain on Saturday. What is the probability that it will **NOT** rain on either of the two days?

40% no rain Friday

80% no rain Saturday

$40\% \cdot 80\% = \boxed{32\%}$

.40
.80
00
3200
.3200
or 32%

33) Write the equation $\log_2 \frac{1}{16} = x$ in exponential form.

switch it! (that's it)

$$\boxed{2^x = \frac{1}{16}}$$

34) Which is the first ***incorrect*** step in simplifying $\log_4 \frac{4}{64}$?

Step 1: $\log_4 \frac{4}{64} = \log_4 4 - \log_4 64$ ✓ good

Step 2: $= 1 - 16$ ← should be 3

Step 3: $= -15$

$\log_4 4$ → $4^1 = 4$ → 1

$\log_4 64$ → $4^3 = 64$ → 3

35) Express $\log_5 50$ as a quotient of common logarithms.

fraction → $\log_{10}$ or log

$$\boxed{\frac{\log 50}{\log 5}}$$

36) What is the value of $\log_3 81$?

$\log_3 81 = x$ → switch it!

$3^x = 81$

$\boxed{x = 4}$

$3 \cdot 3 \cdot 3 \cdot 3$ (4)

$3 \cdot 3 = 9$

$9 \cdot 3 = 27$

$27 \cdot 3 = 81$

37) If $\log 3 \approx 0.477$ and $\log 5 \approx 0.699$, what is the approximate value of $\log 75$?

$\log 75 = \log 3 \cdot 5 \cdot 5$

$= \log 3 + \log 5 + \log 5$

$= 0.477 + 0.699 + 0.699$

$= \boxed{1.875}$

$2 \; 2$

$.477$

$.699$

$.699$

1.875

38) If $\log_2 x = -3$, what is the value of x?

switch it!

$2^{-3} = x$

$\frac{1}{2^3} = x$

$\boxed{x = \frac{1}{8}}$

39) What is the solution to the equation $8^x = 40$? Express your answer as a quotient of common logarithms.

switch it!

$\log_8 40 = x$ ← not done, need common logs!

$\boxed{\frac{\log 40}{\log 8}}$

40) Expand $\ln \frac{a^6}{5b^3}$ as much as possible. (top / bottom)

$= \log a^6 - \log 5b^3$ (top / bottom)

$= \log a^6 - (\log 5 + \log b^3)$

$= 6 \log a - (\log 5 + 3 \log b)$

– or –

$\log a^6 - \log 5b^3$

$= \log a^6 - \log 5 - \log b^3$ ← no parentheses

$= 6 \log a - \log 5 - 3 \log b$

41) Condense the expression: $\log_3 40 - 3\log_3 2 + \log_3 x$

$= \log_3 40 - \log_3 2^3 + \log_3 x$

$= \log_3 40 - \log_3 8 + \log_3 x$ ← left to right

$= \log_3 \frac{40}{8} + \log_3 x$

$= \log_3 5 + \log_3 x$

$= \boxed{\log_3 5x}$

42) Solve for x: $\log_5 (x+4) = 2$] switch it!

$5^2 = x + 4$

$25 = x + 4$

$-4 \quad -4$

$\boxed{21 = x}$

43) Solve for x: $\log_2 7 + \log_2 x = 4$

$\log_2 7x = 4$] switch it!

$2^4 = 7x$

$\frac{16}{7} = \frac{7x}{7}$

$\boxed{x = 16/7}$